



BASIC MATHEMATICS

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A refresher on the important skills you will need before you can Start Algebra. This can be used as a self-teaching guide or as study guide. Use it with a textbook (or instructor !) for more practice. Use the blank pages for scraps.



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FINAL

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Evaluate and simplify whenever possible:

$$76 \div 0.01 = \underline{\hspace{2cm}} \quad (0.12)^2 \underline{\hspace{2cm}}$$

$$89.68 \times 10.2 = \underline{\hspace{2cm}} \quad 7/20 \underline{\hspace{2cm}}$$

$$7/9 - 2/3 = \underline{\hspace{2cm}} \quad 6/12 \underline{\hspace{2cm}}$$

$$2/3 + 5/6 + 3/8 \underline{\hspace{2cm}} \quad (2 \frac{1}{2}) (3 \frac{1}{4}) \underline{\hspace{2cm}}$$

$$7/4 \div 1/2 \underline{\hspace{2cm}} \quad 5/7 \times 28/15 \underline{\hspace{2cm}}$$

$$0.07385 \times 1,000 \underline{\hspace{2cm}} \quad 4 \frac{2}{3} - 2 \frac{5}{6} \underline{\hspace{2cm}}$$

$$1.6 + 3.24 + 9.8 \underline{\hspace{2cm}} \quad 26,643.9 / 100 \underline{\hspace{2cm}}$$

$$12.88 / 0.056 \underline{\hspace{2cm}} \quad 7.0046 - 3.0149 \underline{\hspace{2cm}}$$

Simplify $16/18 \underline{\hspace{2cm}}$ $48/36 \underline{\hspace{2cm}}$ $100/25 \underline{\hspace{2cm}}$

Write as an improper fractions: $6 \frac{3}{7} \underline{\hspace{2cm}}$ $2 \frac{5}{7} \underline{\hspace{2cm}}$

Write as a mixed number: $3.2 \underline{\hspace{2cm}}$ $105/9 \underline{\hspace{2cm}}$

Write as a percent $0.073 \underline{\hspace{2cm}}$ **Write as a decimal** $195.6\% \underline{\hspace{2cm}}$

What is 2% of 16.8 ? $\underline{\hspace{2cm}}$ **30 is 25% of what number ?** $\underline{\hspace{2cm}}$

40 is 20 % of what number ? $\underline{\hspace{2cm}}$ **What is 3.5% of 180 ?** $\underline{\hspace{2cm}}$

What is 30 percent of 30 ? $\underline{\hspace{2cm}}$ **7/8 expressed as a percent is ?** $\underline{\hspace{2cm}}$

Solve for x: (solve the proportions)

$$2/9 = x/27 \underline{\hspace{2cm}} \quad 5/8 = x/48 \underline{\hspace{2cm}}$$

$$80/14 = 20/x \underline{\hspace{2cm}} \quad x/6 = 15/30 \underline{\hspace{2cm}}$$

word problems:

Sam's soccer team played 160 games and won 65% of them. How many games did Sam's team win ?

In a survey of 80 people, $1/2$ of the people chose desert as their favorite food, $1/4$ of them like to eat milk chocolate. How many people in the survey like milk chocolate ?

Your gas tank was $7/8$ full. After you drives 150 miles, the tank was $1/3$ full. What fraction of a tankful of gas did she use on her trip ?

A recipe for $3 \frac{1}{2}$ cups of salsa calls for 7 tomatoes. If Richard wants to increase the amount of salsa to $4 \frac{3}{4}$, how many tomatoes will he need ?



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A computer has 2.0 gigabytes (GB) of memory. If the system software takes 1.1 GB and other applications take 0.7 GB, how much memory is left ?

What is the minimum amount of fabric that you must buy to make 2 pairs of shorts if you need exactly $1\frac{3}{8}$ of fabric for each pair ?

If the average of 2, 7 and N is 4, what is N ? Choose an answer : 3 ? $4\frac{1}{3}$? 5? 13 ?

A train leaves at 3:23PM and arrives at its destination at 5:19PM. How many minutes did it take to make the trip ?

If 6 students in a class of 20 students are seniors, what percent of the class are seniors ?

In an angle of 360° measures 6,400 miles, how many degrees are in an angle of 1,600 miles ?

A 4-inch stack of computer chips is on the table. Each computer chip is $\frac{2}{9}$ of an inch. How many computer chips are in the stack ?

Emma is laying wall tiles in the kitchen. Each tile covers $3\frac{1}{2}$ square inches of space. They plan to cover 210 square inches of wall space. How many tiles will they need ?



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REVIEW

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You sold 2,941 bushels of oranges for a total of 58,231. Which is the best estimate of the price per bushel ?

An instrument store gives a 10% discount to all students off the original cost of an instrument. During a back to school sale an additional 15% is taken off the discounted price. Julie, a student at the local high school, purchases a flute for \$306. How much did it originally cost ?

What is the ratio of a 15° angle to a right angle ? (right angle = 90) _____

What is "\$1.92 for 8 peaches" written as a unit rate ? _____

The property tax on a \$75,000 home is 41,875. At this rate, what is the property tax on a home worth \$125,000 ?

What is the ration of a side of a square to the perimeter equals 9? _____

Write the comparison 9 teaspoons to 4 teaspoons as a ratio ? _____

If you buy 3 pairs of pants for \$45.00, how much would 2 pairs cost ? _____

Write "120 miles on 8 gallons" as a rate in simplest form _____

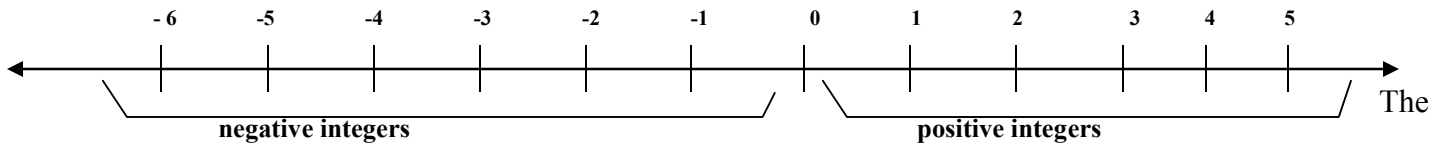
A bank demands a loan payment of \$10.75 each month for every \$1,000 borrowed. At this rate, what is the monthly payment for a \$25,000 loan ?



INTEGERS

NUMBER LINE OPPOSITES

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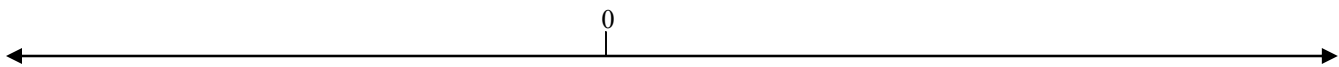


numbers below zero are called negative integers and the numbers above are called positive integers. Each number has an opposite. The **opposite** of 3 is - 3 . We have $3 + (-3) = 0$. 0 is the **identity element** for addition. The opposite is also called **additive inverse**. -3 and 3 are additive inverses.

Here is a list of integers: +7 , -3, -1 , 5, -4 , 6 , -2 , -8,

A) Write the opposite of each integer _____

B) Graph the integers and their opposites on the number line:



C) Write the integers from least to greatest

Compare. Write > or <.

Example $-3 < 1$ and $-10 < 3$ and $-199 < -22$. Remember “more negative” is a number, smaller it is.

-11		-31		11		-31		-11		31		11		31		-56		56		0		-3
-----	--	-----	--	----	--	-----	--	-----	--	----	--	----	--	----	--	-----	--	----	--	---	--	----

Write an integer to represent each situation

Example I lost 10 lb. Integer = -10 I make \$4/hour . Integer = +4 or 4

200 feet below sea level _____

3 km high mountain (above sea level) _____

You win \$100 at the lottery _____

You buy a \$30 gift for your friend _____

20 F below zero _____

100 F above zero _____

Your score is up 3 points _____

Your score is down 3 points _____

You go down 3 floors _____

You have a debt of \$50 _____

Loss of 10 yards _____

Up 10 floor _____



INTEGERS

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The **absolute value** of a number is the distance from 0. The absolute value of -3 is 3 (it is located 3 units from 0). We write: $|-3| = 3$. The absolute value takes away negative signs. $|-3,200| = 3,200$. It has no effect on positive integers: $|233| = 233$

Find each absolute value

Example $|-33| = 33$

$|-1|$ _____ $|0|$ _____ $|56|$ _____ $|-23|$ _____ $|-10|$ _____

You have \$500 on you account. You spend \$600. Your balance is now - \$100 or negative \$100. We write: **$\$500 + (- \$600) = - \$100$** . Subtract 500 from 600 and give the difference a negative sign. When adding integers with opposite signs, subtract their absolute values and give the difference the sign of the larger number. Examples: $3 + (-4) = -1$ but $-3 + 4 = 1$

You have a negative balance of \$100 or -\$100. You spend \$300 more. Your balance is even more negative. Your balance is: - \$400. We write $-\$100 + (-\$300) = - \$400$. Add \$100 and \$300 and give the sum the common sign. When adding integers with same sign, add their absolute values and give the sum the common sign. Examples: $-3 - 4 = - 7$ but $3 + 4 = 7$

Find the sum of integers.

Example $-3 + -2 = -5$; $-3 + 2 = -1$; $2 + -3 = -1$; $-3 + 3 = 0$; $-3 + 0 = -3$

$300 + -200$	$300 + 200 =$	$300 + 200 =$	$200 + - 300 =$	$-200 + 300 =$
$600 + -700 =$	$600 + 700 =$	$- 600 + - 700 =$	$- 700 + 600 =$	$-600 + 700 =$
$0 + - 20 =$	$- 20 + 0 =$	$- 20 + 20 =$	$200 - 200 =$	$- 20 + -20 =$

To subtract integers, add the opposite (or additive inverse) of the number to be subtracted
Examples: $3 - 4 = 3 + (-4) = - 1$ and $3 - (-4) = 3 + (4) = 7$ **opposite of 4 is - 4 and opposite of -4 is 4**

Find the difference or sum

Example $1 - (-1) = 1 + (1) = 2$ or $1 - (1) = 1 + (- 1) = 0$ or $-1 - (-1) = -1 + (1) = 0$ or $-1 - 1 = -1 + (-1) = -2$

$8 - (-7) =$	$-8 - 7 =$	$-8 + (-7) =$	$-8 - 7 =$	$8 + (-7) =$	$-8 + 7 =$
$-3 - (-3) =$	$-3 - 3 =$	$-3 + (-3) =$	$3 - 3 =$	$-3 + 3 =$	$3 + (-3) =$
$-4 - (-2) =$	$4 - 2 =$	$2 - 4 =$	$-2 - 4 =$	$-2 - (-4) =$	$-2 + (-4) =$
$-10 - 10 =$	$-10 + (-10) =$	$10 - 10 =$	$10 - (-10) =$	$-10 - (-10) =$	



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Remember the rules:

$2 - (-3) = 2 + 3 = 5$	<i>minus and minus makes plus</i>	$- (-$ becomes $+$
$2 - (3) = 2 - 3 = -1$	<i>minus and plus makes minus</i>	$- (+$ becomes $-$
$2 - (+3) = 2 - 3 = -1$	<i>minus and plus makes minus</i>	$- ($ becomes $-$
$2 + (-3) = 2 - 3 = -1$	<i>plus and minus makes minus</i>	$+ (-$ becomes $-$

Add or Subtract. Remember *minus and minus makes plus.*

$- (-$ is $+$

Example: $-2 - (-3) - 2 = -2 + 3 - 2 = 1 - 2 = -1$

$2 + (-3) - 5 =$	$-3 - (4) - (-3) + 2 =$	$5 - 6 + (-4) - 5 =$
$-2 - (-3) + (-5) - (3) =$	$4 - (-4) - 4 =$	$7 - (-2) + 7 - 10 + (-10) =$
$-4 - 4 - 4 - 4 =$	$0 - (-0) + 0 =$	$1 - 1 + 1 - 1 - (-1) + 1 - (+1) =$
$-44 - (-33) =$	$89 - 17 =$	$-75 - 23 =$
$65 - (-45) =$	$100 - (-71) =$	$-8 + 25 =$

Review: Add or subtract from left to right

Example: $|-1| - |-1| = 1 - 1 = 0$ and $|-1 - 1| = |-2| = 2$

$|-10 - 10| =$ _____ $|-10 - (-3) + 1| =$ _____

$|-10 - (-10)| =$ _____ $|-10| - |-10| =$ _____

Review: write the additive inverse of each integers

-18 _____ 12 _____ -36 _____ -1 _____

Review: order integers from least to greatest

-33, 180, -100, 0, -11, -342, 10 _____

Solve Remember a temperature below zero or a depth below sea level or a debt is represented by a negative integer.

A scuba diver was 10 feet deep (below 0).

He ascended 4 feet and then descended 6 feet. What was his depth then? _____

The temperature this morning was -2F. By the evening the temperature dropped 5F. Then the temperature dropped another 2F. What was the temperature then? _____

Your balance is - \$2000 (negative) .You spend \$5000 today but then deposit \$7,500.

What is your new balance? _____

The temperature in a city is -3F when it was 10F in another city. What was the difference in temperature between those cities? _____

You have \$500 in your checking account. You deposit a wage check of \$300. You write a check in the amounts \$150, \$600, \$250. After these transaction what is your balance? _____



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The product or quotient of integers with like signs is positive and with unlike signs is negative.
Examples: $-2 \times 3 = -6$ (unlike signs) ; $-2 \times -3 = +6 = 6$ (like signs) ; $3 \times -2 = -6$ (unlike signs)
When multiplying or dividing first figure the sign out and then multiply or divide the numbers.

Find the product or quotient

Example: $3 \times -3 = -9$; $-9 \div 3 = -3$; $-3 \times -3 = 9$; $-9 \div -3 = 3$; $-3 \times -1 = 3$; $-9 \div -1 = 9$; $0 \div -3 = 0$

$-4 \times -5 =$	$4 \times -5 =$	$-4 \times 5 =$	$-4 \times 0 =$	$-4 \times -1 =$	$0 \div -4 =$
$10 \div -5 =$	$-10 \div 5 =$	$-10 \div -5 =$	$-10 \div -1 =$	$0 \div (-10) =$	$-10 \div 2 =$

You can write a multiplication as $2 \cdot 3$ or $2(3)$ instead of 2×3
Remember multiplication is **commutative**: $2(-3) = -3(2) = 3(-2) = -6$ The sign are unlike, the product is -
You can write a division $9 \div 3$ as a **fraction** : $\frac{9}{3}$ or $9/3$. 9 is the **numerator** and 3 the **denominator**.

Find the product or quotient. Watch the signs.

$3(-2) = 3 \times -2 = -6$	$2 \cdot 3 =$	$(-2) \cdot 3 =$	$4(-2) =$
$(-4)(-2) = -4 \times -2 =$	$-4/2 = -4 \div 2 =$	$-4/-2 =$	$4/-2 =$

If you have more than one negative sign, follow the rules: odd number of negative signs, the product or quotient is negative. Even number, the quotient is positive. First figure out the sign and then multiply.
Examples: $(-1)(-1)(-3) = -1 \times 1 \times 3 = -3$ there are 3 negative signs. The product is - (3 is odd)
 $(-1)(-2)(-1)(-1) = +1 \times 2 \times 1 \times 1 = +2 = 2$ there are 4 negative signs. The product is + (4 is even)

Find the product

Example: $(-2)(3)(-1) = +2 \times 3 \times 1 = +6$

$-2(-3)(-1) =$	$-2(3)(-1) =$	$-2(-3)(2) =$	$-2(+2)(-2)(-2) =$
$(-3)^3 = (-3)(-3)(-3) =$	$(-3)^2 = (-3)(-3) =$	$(-2)^3 =$	$(-2)^4 =$

Order of operations: divide or multiply first (from left to right) before you add or subtract (from left to right).
Example: $2 - (-2)(-1)(-3) = 2 + 2 \times 1 \times 3 = 2 + 6 = 8$.

Multiply and add or subtract

Example $-1 + (-1)(-1)(-1) = -1 - 1 = -2$; $-2 + (-3)(0) = -2 + 0 = -2$

$2 - (-2)(-3)(-1) =$	$-2 + (-3)(2)(-5) =$	$2 - (-3)(-4)(-1)(-2) =$
$4 + (-3)(0)(-1) =$	$3 - (6)(-6)(-6) =$	$(-2)(-3) - 2(-3) - 10 =$
$-7 - (10)(-10)(-10)(-10)(10)(-10) =$	$5(-2) + (-2)(-2) =$	



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To evaluate an expression, follow the rule:

1. evaluate the exponential expressions
2. divide or multiply in order from left to right
3. subtract or add in order from left to right

Example: $2 + 3 \cdot 2^2 + 4 \div 2 = 2 + 3 \cdot 4 + 4 \div 2 = 2 + 12 + 4 \div 2 = 2 + 12 + 2 = 16$

Evaluate the expressions

Example: $5^2 (-5) \div 5 - 10 = 25 (-5) \div 5 - 10 = -50 \div 5 - 10 = -10 - 10 = -20$

Exponent first multiply divide subtract

$15 \div 3 + 3(-2)$ _____

$2 + 15 \div 3(2)$ _____

$8 \cdot 3^2 - 3$ _____

$48 \div 6 - (3^3)$ _____

$8 \div 2 \times 2 \times 2 \times 2 + 3(-1) = 4 \times 2 \times 2 \times 2 + 3(-1)$ _____

$10 \div 2 \cdot 2 \cdot 2 + 1 - 2^2$ _____

$2(-10) \div 22 + 3 \cdot (-4) + 1$ _____

If an expression has grouping symbols lie () or { } or [] , compute inside the grouping first following the order of operations described previously. Compute from the inside groupings to the outside groupings. Only when there is no more grouping can you apply once more the previous rules.

Example: $3(3 + 2) - 3^2 \div -3 = 3 \cdot 5 - 3^2 \div -3 = 3 \cdot 5 - 9 \div -3 = 15 - 9 \div -3 = 15 + 9 \div 3 = 15 + 3 = 17$

Add inside the parentheses exponent multiply sign of the quotient divide add

Evaluate the expressions

Example: $[2(14 + 8)] + 12(-2) = (2(22)) + 12(-2) = 44 + 12(-2) = 22 - 24 = -2$

$20 \div 4 + \{ 2 \cdot 3^2 - [3 + (6 - 2)] \}$ _____

$20 \div 5 + \{ 3 \cdot 2^2 - [4 + (4 - 1)] \}$ _____

$-10(2^2 - 2) + 1$ _____ $4 - 12 \div 3 - 9$ _____

$| 3(-5) \div (9+6) |$ _____ $-(-(-(-(-1))))$ _____

Review. Write in scientific notation:

3,000 _____ 2,000,000 _____ 15,000 _____

Evaluate:

33×10 _____ 33×100 _____ $33 \div 10$ _____ $33 \div 100$ _____ $3.33 \div 100$ _____

PRE-ALGEBRA

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Group of numbers

- The number 1, 2, 3, 4, 5 Are called **natural numbers** or **counting numbers**.
- The numbers 0, 1, 2, 3, 4 Are called **whole numbers**.
- The numbers ...-2, -1, 0, 1, 2, ... are called **integers**. It includes negative and positive integers
- fractions $\frac{3}{2}$, $\frac{7}{8}$.. Are called **rational numbers**. Since $5 = \frac{5}{1}$, integers are rational numbers. Decimals and repeating decimals (0.3454545...) are also rational numbers.
- $\sqrt{3}$, π , $\sqrt{5}$ Are **irrational numbers**

Commutative law for addition : $2 + 3 = 3 + 2$ order of the *addends* doesn't make a difference.

Associative law for addition : $(2 + 3) + 4 = 2 + (3 + 4)$ *groupings* doesn't make a difference.

The **identity element** for addition is 0 : $3 + 0 = 3$

The **additive inverse** of any number is the **opposite** of the number.

A number + additive inverse = identity element : $3 + (-3) = 0$

Ways to show multiplication: 4×3 (4) (3) (4) 3 $4 \cdot 3$ 4 (3)

$12 = 3 \times 4$ 3 and 4 are called **factors** of 12.

$12 = 2^2 \times 3$ 2 and 3 are called **prime factors** of 12 $2^2 \times 3$ is called the **prime factorization** of 12.

Ways to show division: $12 \div 3 = 12/3 = 1/3 \times 12$ *one third of 12 = 12 divided by 3*

Commutative law for multiplication : $2 (3) = 3 (2) = 6$ order of *factors* doesn't make a difference.

Associative law for multiplication: $(2 \times 3) \times 4 = 2 \times (3 \times 4)$ *groupings* doesn't make a difference.

The **identity element** for multiplication is 1: $3 \times 1 = 1 \times 3$

The **multiplicative inverse** is the reciprocal of the number. The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

A number x its reciprocal = identity element. $\frac{2}{3} \times \frac{3}{2} = 1$

Multiplication is **distributive** over addition and subtraction. $2 (3 + 4) = 2(3) + 2 (4)$

4^5 means that 4 is to be used as a factor five times. 5 is the exponent and 4 the base.

It is read as *four raised to the fifth power* : $4 \times 4 \times 4 \times 4 \times 4$

$4^{-5} = \frac{1}{4^5}$ if the exponent is negative, then the base and exponent may be dropped under the 1 in a fraction

To remove the sign. $2^{-3} = \frac{1}{2^3}$

$4 \times 4^2 \times 4^5 = 4^8$ keep the base, add the exponents

$(4^2)^3 = 4^6$ keep the base and multiply the exponents.

Order of operations: parentheses / power and square roots / multiplication or division which ever comes first

From left to right / add or subtract which ever comes first from left to right

Try: $10 - 3 \times 6 + 10^2 + (6 + 1) \times 4$

answer = 120