<u>conservative Field - gradient Field -</u>

<u>or how to avoid line integrals.</u>

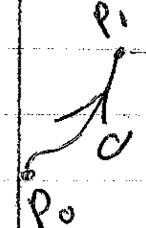
$\vec{F}$ is a conservative Field or gradient Field if $\vec{F} = \vec{\nabla} f$.

$f(x,y)$ is called the potential and is defined up to a constant.

(In Physics we write $\vec{F} = -\vec{\nabla} f$).

In that case, we can simplify the line integral using the Fundamental theorem of calculus.

$$\int_C \vec{F} \cdot d\vec{r} = \int_C \text{grad} f \cdot d\vec{r} = \int_C \vec{\nabla} f \cdot d\vec{r}$$



$$\Rightarrow \boxed{\int_{P_0}^{P_1} \vec{\nabla} f \cdot d\vec{r} = f(P_1) - f(P_0)}$$

<u>proof</u>

$$x = x(t) \quad dx = x'(t)dt$$
$$y = y(t) \quad dy = y'(t)dt$$

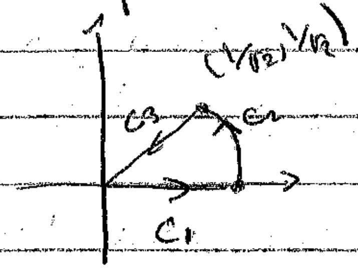
$$\int_C \vec{\nabla} f \cdot d\vec{r} = \int_C f_x(t)\, dx + \int_C f_y\, dy = \int_{t_0}^{t_1} \left(f_x \frac{dx}{dt} + f_y \frac{dy}{dt}\right) dt$$

$$\int_{t_0}^{t_1} \frac{df}{dt}\, dt = f(P_1)\ f(P_0).$$

Example $\vec{F} = \langle y, x \rangle$

$\vec{F} = \vec{\nabla} f \Rightarrow f(x,y) = xy$

$\vec{F}$ is a gradient field, so



$$\int_{C_2} \vec{F} \cdot d\vec{r} = f(1/\sqrt{2}, 1/\sqrt{2}) - f(1,0)$$

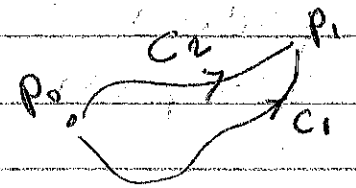
$$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} - 0 = \frac{1}{2}$$

$$\int_{C_1} \vec{F} \cdot d\vec{r} = f(1,0) - f(0,0) = 0$$

$$\int_{C_2} \vec{F} \cdot d\vec{r} = f(0,0) - f\left(\frac{1}{\sqrt{2}}, 1/\sqrt{2}\right) = -\frac{1}{\sqrt{2}}$$

} we get the same thing

* If the field is conservative, the line integral does not depend on path = path independence.



$$\int_{C_1} \vec{\nabla} F \cdot d\vec{r} = \int_{C_2} \vec{\nabla} F \cdot d\vec{r}$$

* Also the line integral on a closed path = 0

$$\int_{C_1} \vec{\nabla} \vec{F} \cdot d\vec{r} - \int_{C_2} \vec{\nabla} F \cdot d\vec{r} = \oint_C \vec{F} \cdot d\vec{r} = 0$$

* $M\,dy + N\,dy = f_x\,dy + f_y\,dy = df$ = exact differential.