<u>First order, linear diff equation.</u>

$y' + p(t)\,y = q(t)$ is solve by using the integrating factor $u = \int p(t)\,dt$

2 particular equations are of interest:

① $y' + ky = k\,q(t)$ with $k > 0$

② $y' + ky = q(t)$ with $k > 0$

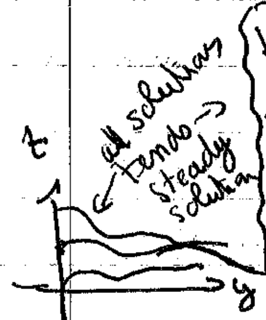
$q(t)$ is the input (exitation) and $y(t)$ the response.

the <u>general solution</u> looks like:

$$\boxed{\underbrace{A e^{-kt} \int e^{kt} q(t)\,dt}_{} + \underbrace{B e^{-kt}}_{}}$$

this is the steady state part of the solution.

if $t \to \infty$ this part $\to 0$ it is called the transcient solution. depends on the initial conditions.



solution of ② is $k\, e^{-kt} \int q(t) e^{kt}\,dt + C e^{-kt}$

<u>Example:</u> if $q(t) = \cos \omega t$

the solution is $\dfrac{1}{\sqrt{1 + \left(\frac{\omega}{k}\right)^2}} \cos(\omega t - \phi)$

(steady part)

if $k < 0$ (Biology economics)

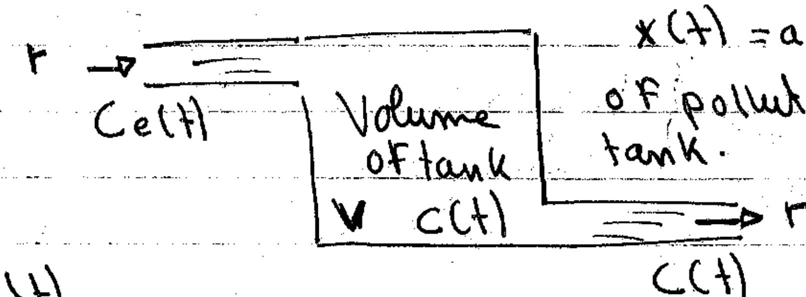
the general solution

$a > 0$ becomes $y = \underbrace{e^{at} \int q(t)\, e^{-at}\, dt + C e^{at}}$

there is no transcient solution any more, $C e^{at}$ becomes the dominant part.

<u>Example</u>: Mixing problem.

some polluted Fluid, with $C_e(t)$ the pollutant concentration, gets in a tank at a rate r. The Fluid leaves the tank and the concentration of the pollutant is $C(t)$. the leaving rate is the same.



$x(t)$ = amount of pollutant in tank.

Find $x(t)$.

solution: $\frac{dx}{dt} = C_e(t)\, r - C(t)\, r$

$x = C * V$

$\frac{dc}{dt} V = C_e r - C r \Rightarrow \boxed{C' + \frac{C}{V} r = \frac{C_e r}{V}}$

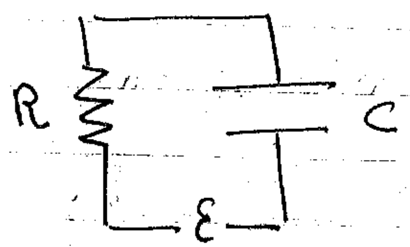
this equation is of the type $\boxed{y' + hc = h\,p(t)}$

$P_2 > 0$

q(t) could be sinusoidal if the concentration varies like a sine function (more polution in the morning then decreases through day and night and increases again)

P_2 large if r large or V small, in that case the response $C(t)$ closely follows the input $C_e(t)$ and the transient solution goes to 0 rapidly.

Example 2 RC circuit



$q(t)$ = charge on capacitor

$\frac{dq(t)}{dt} = i(t)$ = current in circuit

$U_C = Q/C \qquad U_R = Ri$

$$\mathcal{E}(t) = R\frac{dq}{dt} + q/C \Rightarrow \boxed{q' + \frac{q}{RC} = \frac{\mathcal{E}(t)}{R}}$$

type ① equation: $y' + hy = p(t)$

if $\mathcal{E}(t) = A\cos\omega t$ solution $= \frac{A}{R\sqrt{\omega^2 + 1/R^2C^2}}\cos(\omega t - \phi) + Ce^{-t/RC}$

steady solution →