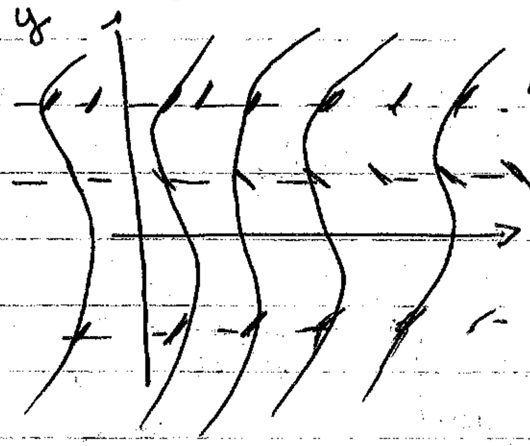
$\frac{dy}{dt} = F(y)$ qualitative study.

↳ not time dependant.

atonomous equations.

you could separate variable but here is away to sketch the behavior of the solutions without solving the equations.

For a given y, the slope is constant. the integral lines are horizontal line.



given slope, y = constant

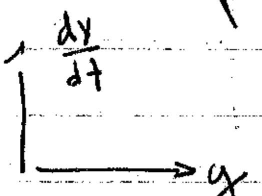
all the solutions are shifted relative to each other.

critical points are defined by : $\frac{dy}{dt} = F(y_0) = 0$
the slope = 0 for a given y_0.

follow steps to sketch:

① Find the critical points: solve $F(y_0) = 0$

② trace



to see for which values of y, $\frac{dy}{dt} > 0 \Rightarrow y$ ~~increases~~ increases.
for which values of y, $\frac{dy}{dt} < 0 \Rightarrow y$ decreases.
$y = y_0$ critical point.

(3) sketch the solutions $y(t)$

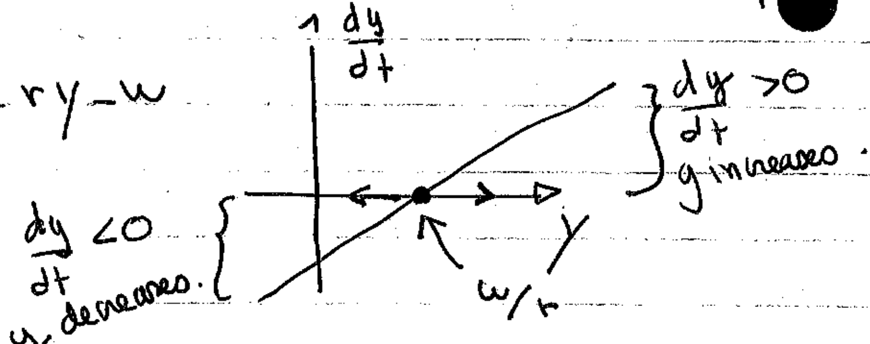
Example 1 : money in your account being embezzled

- y : money in the bank account
  r = (continuous) is the interest rate
  w = rate of embezzlement (money sneaked out)

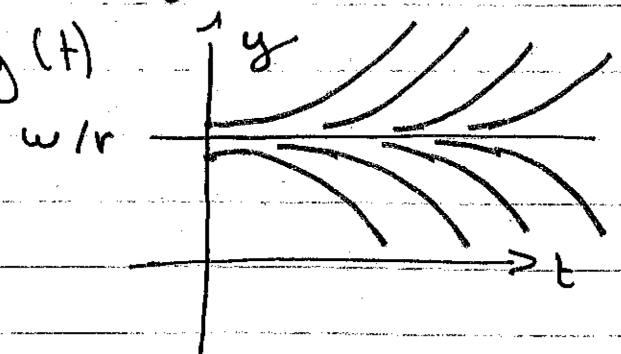
$$\boxed{\frac{dy}{dt} = ry - w}$$

(1) solve $ry - w = 0 \Rightarrow y = w/r$ is a critical point

(2) trace $f(y) = ry - w$



(3) sketch $y(t)$



solutions are repelled by $y = w/r$ unstable critical point.

so if at $t=0$ the money is greater than w/r in your account, then the money increases.
if at $t=0$ $y < w/r$, the money y decreases!

Example 2 logistic equations
describe how population increases / population behavior $y(t)$.

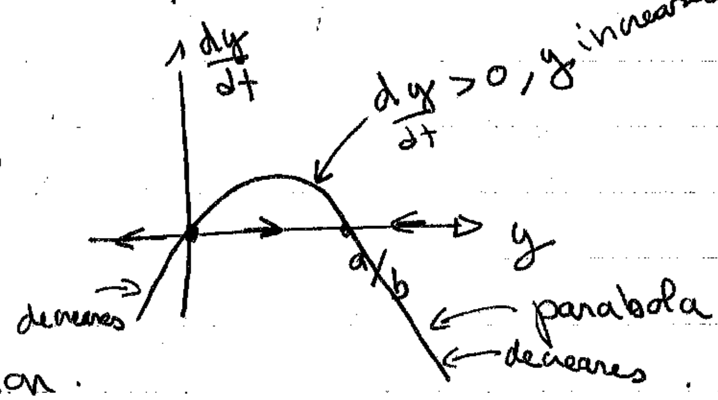
$$\boxed{\frac{dy}{dt} = k y}$$ ← if k = constant, we deal with simple exponential growth.

but in reality, the population growth is limited by available ressources so
$k = a - by$

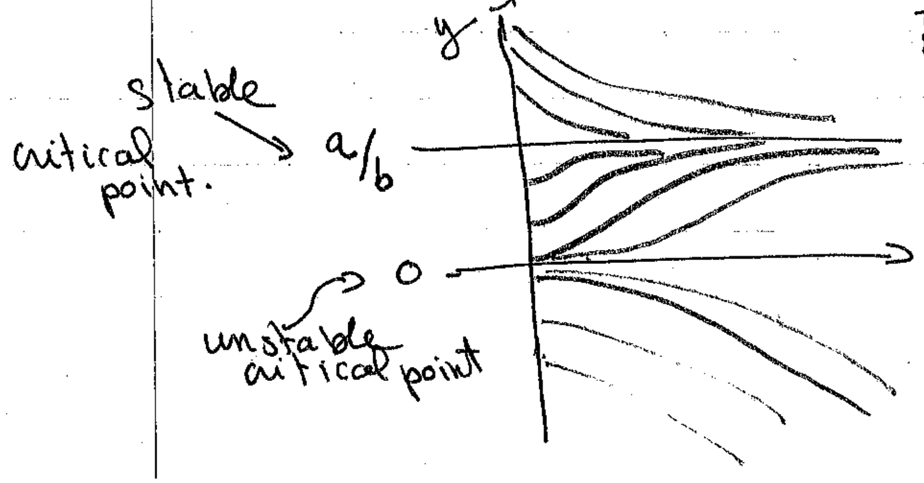
$$\boxed{\frac{dy}{dt} = (a-by)y}$$ ← logistic equation
growth factor is linear
$k = a - by$

① critical points $(a-by)y = 0$
$y = 0$ and $y = a/b$.

② trace $\frac{dy}{dt}$ vs y



③ sketch the solution.



} $t=0$, $y > a/b$,
} $y \to a/b$.

} if $t=0$, $y>0$ and $y<a/b$
} population grows and reaches a Max $= a/b$.

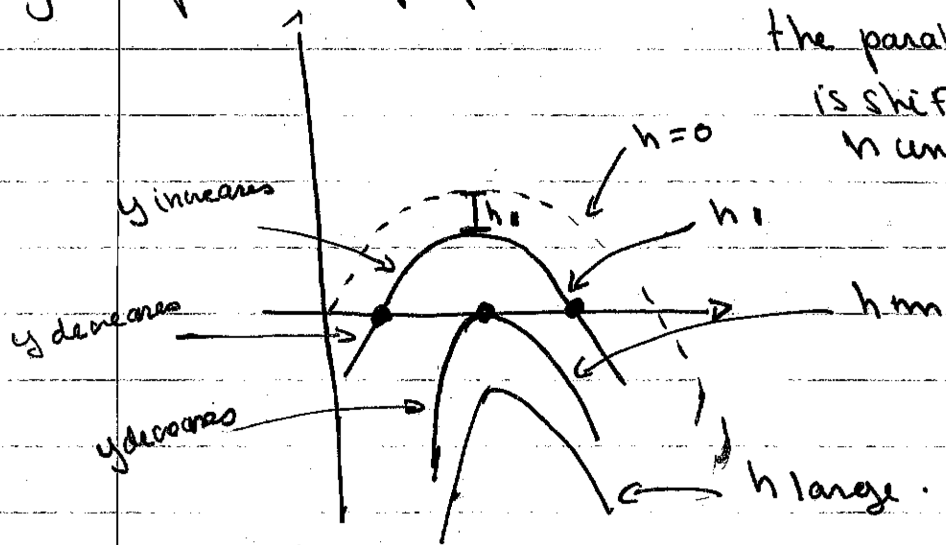
} does not mean any thing, population can't be negative

Example 3: logistic equation with harvesting. Like before but now a constant numbers of members is "harvested" from the population. (Salmon Farms, some salmons are taken from the tank to be eaten).

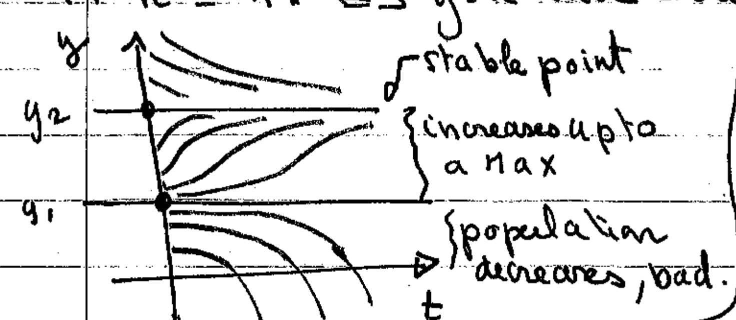
$$\boxed{\frac{dy}{dt} = ay - by^2 - h}$$

don't solve the quadratic equation - use your previous graph.

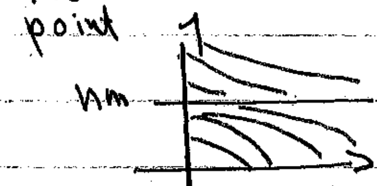
the parabola $ay - by^2$ is shifted vertically h units down.



if $h = h_1 \Rightarrow$ you have 2 critical points.



For $h = h_m \Rightarrow$ 1 critical point



so $y > h_m$ otherwise, the farm is out of business

~~Soz~~ So initial population has to be greater than y_1, otherwise all salmons will be gone.

For h large } the population decreases!