Inhomogeneous 2nd order equation.

$y'' + p(x)y' + q(x)y = f(x)$

$\hookrightarrow$ sometimes called input / signal / driving term Forcing term.

$y(x)$ is called the $\begin{cases}\text{response}\\ \text{output.}\end{cases}$

$y'' + p(x)y' + q(x)y = 0$ is called the $\begin{cases}\text{associated}\\ \text{corresponding}\end{cases}$ equation.

or the reduced equation.

solution is $y_c = C_1 y_1 + C_2 y_2$ where y_1, y_2 are any independent solutions and C_1 and C_2 are arbitrary constant.

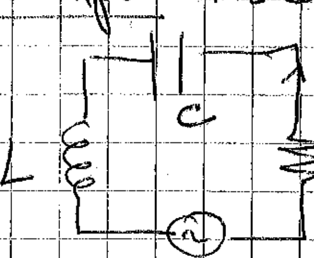
Example 1 (models) $mx'' + bx' + Kx = F(t)$ spring - mass - damping

derived from Newton's 2nd law $F = ma$

$mx'' = -Kx - bx' + F(t)$

spring constant, damping Force, driven Force external Force

Example 2 RLC



Sum of voltage drops $= 0$

$i = q'$

$Li' + Ri + \frac{q}{C} = \mathcal{E}(t)$

$\boxed{Li'' + Ri' + \frac{i}{C} = \mathcal{E}'(t)}$

} this equation governs the flow of current in the circuit.

we can write $y'' + p(x)y' + q(x)y = f(t)$

as $\boxed{Ly(t) = f(t)}$

$\hookrightarrow$ Linear operator 2nd order

The solution is $y_p + y_c$ → solution of the homogeneous equation $y'' + p(x)y' + q(x)y = 0$
y_c = complementary solution.

y_p ← particular solution.

Analogy with first order solution:

$$y = \underbrace{e^{-kt}\int q(t)e^{kt}\,dt}_{\text{particular solution}} + \underbrace{Ce^{-kt}}$$

↳ complementary solution because it depends on initial solution.

if $k > 0$

$$y = \text{steady state} + \text{transcient}$$

steady state ↕ important part.

transcient: $y_c \to 0$, $t \to \infty$.

if $k < 0$ the ce^{-kt} is dominant part.

Question: under what conditions $c_1 y_1 + y_2 c_2 \underset{t \to \infty}{\to} 0$?

$$y = y_p + \underbrace{c_1 y_1 + c_2 y_2}.$$

↳ use the initial conditions.

consider $y'' + Ay' + By = f(t)$

we need to find the conditions for $c_1 y_1 + c_2 y_2 \to 0$, $t \to \infty$.

It depends on the characteristic roots.

characteristic roots	solutions	stability conditions.
real and distinct $r_1 \neq r_2$	$c_1 e^{r_1 t} + c_2 e^{r_2 t}$	$r_1 < 0$, $r_2 < 0$
$r_1 = r_2$ real	$(c_1 + c_2 t) e^{r_1 t}$	$r_1 < 0$
$r = a \pm bi$ complex	$e^{at}(c_1 \cos bt + c_2 \sin bt)$	$a < 0$

ODE is said stable if the characteristic roots have negative real parts.