differential equations.

I. First order linear equation.

standard Form: $y' + p(x)y = q(x)$

to solve Follow the steps:

(1) standard Form

(2) compute the integrating Factor
$u = \int e^{p(x)}\,dx$.

(3) Multiply every thing by u so you get:

$$\underbrace{y'u + (p(x)u)y}_{(uy)'} = q(x)u$$

$$(uy)' = q(x)u$$

with $p(x)u = u'$

or $\frac{du}{dx} = p(x)$

or $\ln u = \int p(x)\,dx$

or $u = e^{\int p(x)\,dx}$.

(4) integrate to solve for y.

Example 1 $x y' - y = x^3$

(1) $y' - \frac{1}{x}y = x^2$

(2) $\int p(x)\,dx = -\int \frac{1}{x}\,dx = -\ln(x)$

$u = e^{-\ln(x)} = \frac{1}{x}$

(3) $\frac{1}{x} y' - \frac{1}{x^2} y = x^2$

$$\left(\frac{1}{x} y\right)' = x$$

(4) $\frac{1}{x} y = \frac{x^2}{2} + C$

$$\boxed{y = \frac{x^3}{2} + Cx}$$

Example 2 $(1+\cos x) y' - \sin x\, y = 2x$

(1) $y' - \frac{\sin x}{1+\cos x} y = \frac{2x}{1+\cos x}$

(2) $\int p(x)\,dx = -\int \frac{\sin x}{1+\cos x}\,dx = \ln(1+\cos x)$

$u = 1+\cos x$

(3) $\left(y(1+\cos x)\right)' = 2x$ ~~(Integrate)~~

$y(1+\cos x) = \frac{2x^2}{2} + C$

$$\boxed{y = \frac{x^2 + C}{1+\cos x}}$$ if $y(0) = 1$ initial conditions

then $1 = \frac{C}{2} \Rightarrow C = 2$